Lecture #3: Rendezvous and connectivity maintenance algorithms

Francesco Bullo[1] Jorge Cortés[2] **Sonia Martínez**[2]

[1]Department of Mechanical Engineering
University of California, Santa Barbara
bullo@engineering.ucsb.edu

[2]Mechanical and Aerospace Engineering
University of California, San Diego
{cortes,soniamd}@ucsd.edu

Workshop on "Distributed Control of Robotic Networks"
IEEE Conference on Decision and Control
Cancun, December 8, 2008

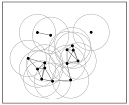
Acknowledgements: Anurag Ganguli, UtopiaCompression

Summary Introduction

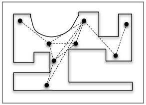
- Motion coordination problem: the rendezvous objective
- Constraining multi-robot motion to maintain connectivity
- Achieving rendezvous under different assumptions on connectivity, for convex and non-convex environments
- Time complexity of rendezvous algorithms under different connectivity assumptions in 1D spaces
- Method of proof is based on a LaSalle invariance principle for set-valued maps

Rendezvous objective

Objective:
achieve multi-robot rendezvous; i.e. arrive at the same location of space

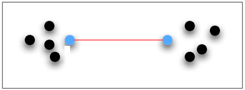


r-disk connectivity



visibility connectivity

But we have to be careful...



Blindly "getting closer" to neighboring agents might lead to disconnection

Network definition and rendezvous tasks

The objective is applicable for general robotic networks
$\mathcal{S}_{\text{disk}}$, $\mathcal{S}_{\text{LD}}$ and $\mathcal{S}_{\infty\text{-disk}}$,
and the relative-sensing networks $\mathcal{S}^{\text{rs}}_{\text{disk}}$ and $\mathcal{S}^{\text{rs}}_{\text{vis-disk}}$

We adopt the discrete-time motion model

$$p^{[i]}(\ell+1) = p^{[i]}(\ell) + u^{[i]}(\ell), \quad i \in \{1,\dots,n\}$$

Also for the relative-sensing networks

$$p^{[i]}_{\text{fixed}}(\ell+1) = p^{[i]}_{\text{fixed}}(\ell) + R^{[i]}_{\text{fixed}} u^{[i]}_i(\ell), \quad i \in \{1,\dots,n\}$$

We usually assume no bound on the control or $u_{\max}$

The rendezvous task

Let $\mathcal{S} = (\{1,\dots,n\}, \mathcal{R}, E_{\text{cmm}})$ be a uniform robotic network
The **(exact) rendezvous task** $\mathcal{T}_{\text{rndzvs}} : X^n \to \{\texttt{true}, \texttt{false}\}$ for $\mathcal{S}$ is

$$\mathcal{T}_{\text{rndzvs}}(x^{[1]},\dots,x^{[n]}) = \begin{cases} \texttt{true}, & \text{if } x^{[i]} = x^{[j]}, \text{ for all } (i,j) \in E_{\text{cmm}}(x^{[1]},\dots,x^{[n]}), \\ \texttt{false}, & \text{otherwise} \end{cases}$$

Suppose that $\mathcal{P} = \{p^{[1]},\dots,p^{[n]}\}$ is the set of agents location in $X \subset \mathbb{R}^d$, P be an array of n points in $\mathbb{R}^d$, and let avrg denote

$$\mathsf{avrg}(\{q_1,\dots,q_k\}) = \frac{1}{k}(q_1 + \dots + q_k)$$

For $\varepsilon \in \mathbb{R}_{>0}$, the **$\varepsilon$-rendezvous task** $\mathcal{T}_{\varepsilon\text{-rndzvs}} : (\mathbb{R}^d)^n \to \{\texttt{true}, \texttt{false}\}$ is

$$\mathcal{T}_{\varepsilon\text{-rndzvs}}(P) = \texttt{true}$$
$$\iff \|p^{[i]} - \mathsf{avrg}\left(\{p^{[j]} \mid (i,j) \in E_{\text{cmm}}(P)\}\right)\|_2 < \varepsilon, \quad i \in \{1,\dots,n\}$$

Outline

Enforcing range-limited links – pairwise connectivity

Pairwise connectivity maintenance problem:
Given two neighbors in the proximity graph $\mathcal{G}_{\text{disk}}(r)$, find a rich set of control inputs for both agents with the property that, after moving, both agents are again within distance r

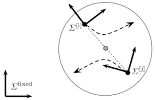
Definition (Pairwise connectivity constraint set)

Consider two agents i and j at positions $p^{[i]} \in \mathbb{R}^d$ and $p^{[j]} \in \mathbb{R}^d$ such that $\|p^{[i]} - p^{[j]}\|_2 \leq r$. The *connectivity constraint set* of agent i with respect to agent j is

$$\mathcal{X}_{\text{disk}}(p^{[i]}, p^{[j]}) = \overline{B}\Big(\frac{p^{[j]} + p^{[i]}}{2}, \frac{r}{2}\Big).$$

Enforcing range-limited links – pairwise connectivity

Note that both robots i and j can independently compute their respective connectivity constraint sets



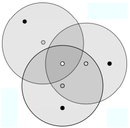
If $\|p^{[i]}(\ell) - p^{[j]}(\ell)\| \le r$, and remain in the connectivity sets,
then $\|p^{[i]}(\ell + 1) - p^{[j]}(\ell + 1)\| \le r$

Enforcing range-limited links – multi-agent connectivity

Definition (Connectivity constraint set)

Consider a group of agents at positions $\mathcal{P} = \{p^{[1]}, \dots, p^{[n]}\} \subset \mathbb{R}^d$. The *connectivity constraint set* of agent i with respect to $\mathcal{P}$ is

$$\mathcal{X}_{\text{disk}}(p^{[i]}, \mathcal{P}) = \bigcap \{\mathcal{X}_{\text{disk}}(p^{[i]}, q) \mid q \in \mathcal{P} \setminus \{p^{[i]}\} \text{ s.t. } \|q - p^{[i]}\|_2 \le r\}$$



Enforcing a less conservative connectivity

Recall definitions of other proximity graphs

Relative neighborhood graph $\mathcal{G}_{\text{RN}}$, the Gabriel graph $\mathcal{G}_{\text{G}}$, and the r-limited Delaunay graph $\mathcal{G}_{\text{LD}}(r)$

The graphs $\mathcal{G}_{\text{RN}} \cap \mathcal{G}_{\text{disk}}(r)$, $\mathcal{G}_{\text{G}} \cap \mathcal{G}_{\text{disk}}(r)$ and $\mathcal{G}_{\text{LD}}(r)$ satisfy:

1. They have the same connected components as $\mathcal{G}_{\text{disk}}(r)$, and
2. they are spatially distributed over $\mathcal{G}_{\text{disk}}(r)$

Consequences are

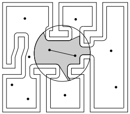
1. Sparser graphs imply fewer connectivity constraints, and
2. agents can determine its neighbors in these graphs

Enforcing range-limited line-of-sight links

Consider a compact nonconvex environment $Q \subset \mathbb{R}^2$ and contract this into $Q_\delta = \{q \in Q \mid \mathsf{dist}(q, \partial Q) \ge \delta\}$ for a small positive δ.

Suppose robots are deployed in Q_δ and constitute a visibility-based network $\mathcal{S}_{\text{vis-disk}}$. That is, j is a neighbor of i iff

$$p^{[j]}(\ell) \in \mathsf{Vi}_{\text{disk}}(p^{[i]}(\ell); Q_\delta) = \mathsf{Vi}(p^{[i]}(\ell); Q_\delta) \cap \overline{B}(p^{[i]}(\ell), r)$$



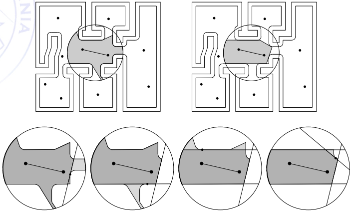
Enforcing range-limited line-of-sight links

The following algorithm computes a sufficient constraint set

function ITERATED TRUNCATION$(p^{[i]}, p^{[j]}; Q_\delta)$
%Executed by robot i at position $p^{[i]}$ assuming that robot j is at position $p^{[j]}$ within range-limited line of sight of $p^{[i]}$

1: $\mathcal{X}_{\text{temp}} := \mathsf{Vi}_{\text{disk}}(p^{[i]}; Q_\delta) \cap \overline{B}\big(\frac{1}{2}(p^{[i]} + p^{[j]}), \frac{r}{2}\big)$
2: **while** $\partial\mathcal{X}_{\text{temp}}$ contains a concavity **do**
3: $\quad v :=$ a strictly concave point of $\partial\mathcal{X}_{\text{temp}}$ closest to $[p^{[i]}, p^{[j]}]$
4: $\quad \mathcal{X}_{\text{temp}} := \mathcal{X}_{\text{temp}} \cap H_{Q_\delta}(v)$
5: **return** $\mathcal{X}_{\text{temp}}$

Enforcing range-limited line-of-sight links



Enforcing range-limited line-of-sight links

Theorem (Properties of the ITERATED TRUNCATION ALGORITHM)

Consider the δ-contraction of a compact allowable environment Q_δ with κ strict concavities, and let $(p^{[i]}, p^{[j]}) \in J$. The following holds:

1. *The* ITERATED TRUNCATION ALGORITHM, *invoked with arguments $(p^{[i]}, p^{[j]}; Q_\delta)$, terminates in at most κ steps; denote its output by $\mathcal{X}_{\text{vis-disk}}(p^{[i]}, p^{[j]}; Q_\delta)$;*
2. *$\mathcal{X}_{\text{vis-disk}}(p^{[i]}, p^{[j]}; Q_\delta)$ is nonempty, compact and convex;*
3. *$\mathcal{X}_{\text{vis-disk}}(p^{[i]}, p^{[j]}; Q_\delta) = \mathcal{X}_{\text{vis-disk}}(p^{[j]}, p^{[i]}; Q_\delta)$; and*
4. *the set-valued map $(p, q) \mapsto \mathcal{X}_{\text{vis-disk}}(p, q; Q_\delta)$ is closed at all $(p, q) \in J$.*

Proof: (Item 3) all relevant concavities in the computation of $\mathcal{X}_{\text{vis-disk}}(p^{[i]}, p^{[j]}; Q_\delta)$ are visible from both agents $p^{[i]}$ and $p^{[j]}$

Enforcing range-limited line-of-sight constraints

Definition (Line-of-sight connectivity constraint set)

Consider a nonconvex allowable environment Q_δ and two agents i and j within range-limited line of sight. We call:

- $\mathcal{X}_{\text{vis-disk}}(p^{[i]}, p^{[j]}; Q_\delta)$ the **pairwise line-of-sight connectivity constraint set** of agent i with respect to agent j
- the **line-of-sight connectivity constraint sets** of agent i with respect to $\mathcal{P}$ is

$$\mathcal{X}_{\text{vis-disk}}(p^{[i]}, \mathcal{P}; Q_\delta) = \bigcap \{\mathcal{X}_{\text{vis-disk}}(p^{[i]}, q; Q_\delta) \mid q \in \mathcal{P} \setminus \{p^{[i]}\}\}$$

Averaging control and communication law

Averaging behavior: move towards a position computed as the average of the received messages

Relation to Vicsek's model for fish flocking and employed to model "opinion dynamics under bounded confidence"

[Informal description:]
At each communication round each agent performs the following tasks: (i) it transmits its position and receives its neighbors' positions; (ii) it computes the average of the point set comprised of its neighbors and of itself. Between communication rounds, each robot moves toward the average point it computed.

The law is uniform, static, and data-sampled, with standard message-generation function

Averaging control and communication law

`Robotic Network:` $\mathcal{S}_{\text{disk}}$ with "discrete-time" motion in $\mathbb{R}^d$,
with absolute sensing of own position, and
with communication range r

`Distributed Algorithm:` AVERAGING
`Alphabet:` $\mathbb{A} = \mathbb{R}^d \cup \{\texttt{null}\}$

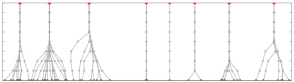
`function` $\text{msg}(p, i)$
1: **return** p

`function` $\text{ctl}(p, y)$
1: **return** $\mathsf{avrg}(\{p\} \cup \{p_{\text{rcvd}} \mid p_{\text{rcvd}} \text{ is a non-null message in } y\}) - p$

Averaging CC law – an implementation in $d = 1$

Note that, along the evolution,

- several robots rendezvous
- some robots are connected at the simulation's beginning and not connected at the simulation's end



Averaging CC law – correctness

Theorem (Correctness and time complexity of AVERAGING law)

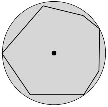
For $d = 1$, the network $\mathcal{S}_{\text{disk}}$, the law $\mathcal{CC}_{\text{AVERAGING}}$ achieves the task $\mathcal{T}_{\text{rndzvs}}$ with time complexity

$$\mathsf{TC}(\mathcal{T}_{\text{rndzvs}}, \mathcal{CC}_{\text{AVERAGING}}) \in O(n^5),$$
$$\mathsf{TC}(\mathcal{T}_{\text{rndzvs}}, \mathcal{CC}_{\text{AVERAGING}}) \in \Omega(n).$$

Circumcenter control and communication laws

Recall the circumcenter definition:

For $X = \mathbb{R}^d$, $X = \mathbb{S}^d$ or $X = \mathbb{R}^{d_1} \times \mathbb{S}^{d_2}$, $d = d_1 + d_2$, **circumcenter** $\mathsf{CC}(W)$ of a bounded set $W \subset X$ is center of closed ball of minimum radius that contains W. **Circumradius** $\mathsf{CR}(W)$ is radius of this ball



Lemma (Properties of the circumcenter in Euclidean space)

Let $S \in \mathbb{F}(\mathbb{R}^d)$. Then, the following holds:

1. $\mathsf{CC}(S) \in \mathsf{co}(S) \setminus \mathsf{Ve}(\mathsf{co}(S))$
2. *if $p \in \mathsf{co}(S) \setminus \{\mathsf{CC}(S)\}$ and $r \in \mathbb{R}_{>0}$ are such that $S \subset \overline{B}(p, r)$, then $]p, \mathsf{CC}(S)[$ has a nonempty intersection with $\overline{B}(\frac{p+q}{2}, \frac{r}{2})$ for all $q \in \mathsf{co}(S)$*

Circumcenter control and communication law

Basic Idea:

- each agent minimizes "local version" of objective function

$$\max\{\|p_i - p_j\| \mid p_j \text{ is neighbor of } p_i\}$$

  i.e., each agent goes toward circumcenter of neighbors and itself (which is the closest point to all these locations)

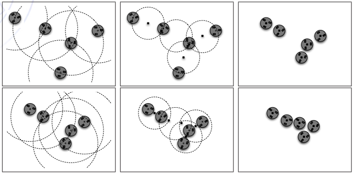
- each agent maintains connectivity by moving inside constraint set

[Informal description:]

At each communication round each agent performs the following tasks: (i) it transmits its position and receives its neighbors' positions; (ii) it computes the circumcenter of the point set comprised of its neighbors and of itself. Between communication rounds, each robot moves toward this circumcenter point while maintaining connectivity with its neighbors using appropriate connectivity constraint sets.

Circumcenter control and communication law

Illustration of the algorithm execution



Circumcenter control and communication law

Formal algorithm description

`Robotic Network:` $\mathcal{S}_{\text{disk}}$ with a discrete-time motion model, with absolute sensing of own position, and with communication range r, in $\mathbb{R}^d$

`Distributed Algorithm:` CIRCUMCENTER

`Alphabet:` $\mathbb{A} = \mathbb{R}^d \cup \{\texttt{null}\}$

`function` $\text{msg}(p, i)$

1: **return** p

`function` $\text{ctl}(p, y)$

1: $p_{\text{goal}} := \mathsf{CC}(\{p\} \cup \{p_{\text{rcvd}} \mid \text{for all non-null } p_{\text{rcvd}} \in y\})$

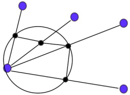
2: $\mathcal{X} := \mathcal{X}_{\text{disk}}(p, \{p_{\text{rcvd}} \mid \text{for all non-null } p_{\text{rcvd}} \in y\})$

3: **return** $\mathsf{fti}(p, p_{\text{goal}}, \mathcal{X}) - p$

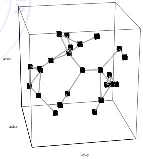
Circumcenter control and communication law

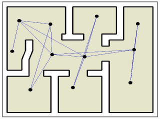
Relaxations:

- Can also be run over any other proximity graph which is spatially distributed over $\mathcal{G}_{\text{disk}}(r)$ or over $\mathcal{G}_{\text{vis-disk},Q_\delta}$
- Bounds can be applied to the control magnitude
- Other alternatives are available where the constraint set is not necessary
  - Use a "parallel circumcenter control and communication law"
  - Use a "1/2 circumcenter algorithm"



Simulations





Correctness

Theorem (Correctness of the circumcenter laws)

For $d \in \mathbb{N}$, $r \in \mathbb{R}_{>0}$ and $\varepsilon \in \mathbb{R}_{>0}$, the following statements hold:

1. *on $\mathcal{S}_{\text{disk}}$, the law $\mathcal{CC}_{\text{CIRCUMCENTER}}$ (with control magnitude bounds and relaxed $\mathcal{G}$-connectivity constraints) achieves $\mathcal{T}_{\text{rndzvs}}$;*
2. *on $\mathcal{S}_{\text{LD}}$, the law $\mathcal{CC}_{\text{CIRCUMCENTER}}$ achieves $\mathcal{T}_{\varepsilon\text{-rndzvs}}$*

Similar result for the parallel circumcenter algorithm and for visibility networks in non-convex environments

Correctness

Theorem (Correctness of the circumcenter laws)

Furthermore, the evolutions of $(\mathcal{S}_{\text{disk}}, \mathcal{CC}_{\text{CIRCUMCENTER}})$, of $(\mathcal{S}_{\text{LD}}, \mathcal{CC}_{\text{CIRCUMCENTER}})$, and of $(\mathcal{S}_{\infty\text{-disk}}, \mathcal{CC}_{\text{PLL-CRCMCNTR}})$ have the following properties:

1. *if any two agents belong to the same connected component at $\ell \in \mathbb{Z}_{\geq 0}$, then they continue to belong to the same connected component subsequently; and*
2. *for each evolution, there exists $P^* = (p_1^*, \dots, p_n^*) \in (\mathbb{R}^d)^n$ such that:*
   1. *the evolution asymptotically approaches P^*, and*
   2. *for each $i, j \in \{1, \dots, n\}$, either $p_i^* = p_j^*$, or $\|p_i^* - p_j^*\|_2 > r$ (for the networks $\mathcal{S}_{\text{disk}}$ and $\mathcal{S}_{\text{LD}}$) or $\|p_i^* - p_j^*\|_\infty > r$ (for the network $\mathcal{S}_{\infty\text{-disk}}$).*

Correctness – Time complexity

Theorem (Time complexity of circumcenter laws)

For $r \in \mathbb{R}_{>0}$ and $\varepsilon \in \,]0,1[$, the following statements hold:

1. *on the network $\mathcal{S}_{\text{disk}}$, evolving on the real line $\mathbb{R}$ (i.e., with $d=1$),* $\mathsf{TC}(\mathcal{T}_{\text{rndzvs}}, \mathcal{CC}_{\text{CIRCUMCENTER}}) \in \Theta(n)$;
2. *on the network $\mathcal{S}_{\text{LD}}$, evolving on the real line $\mathbb{R}$ (i.e., with $d=1$),* $\mathsf{TC}(\mathcal{T}_{(r\varepsilon)\text{-rndzvs}}, \mathcal{CC}_{\text{CIRCUMCENTER}}) \in \Theta(n^2 \log(n\varepsilon^{-1}))$*; and*
3. *on the network $\mathcal{S}_{\infty\text{-disk}}$, evolving on Euclidean space (i.e., with $d \in \mathbb{N}$),* $\mathsf{TC}(\mathcal{T}_{\text{rndzvs}}, \mathcal{CC}_{\text{PLL-CRCMCNTR}}) \in \Theta(n)$.

Results hold for constant comm range, but allow for the diameter of the initial network configuration (the maximum inter-agent distance) to grow unbounded with the number of robots

Extension to visibility network is possible

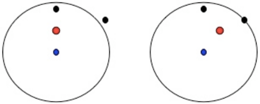
Some bad news...

Circumcenter algorithms are nonlinear discrete-time dynamical systems

$$x_{\ell+1} = f(x_\ell)$$

To analyze convergence, we need at least f continuous – to use classic Lyapunov/LaSalle results

But circumcenter algorithms are discontinuous because of changes in interaction topology



Alternative idea

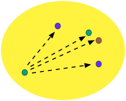
Fixed undirected graph G, define **fixed-topology circumcenter algorithm**

$$f_G : (\mathbb{R}^d)^n \to (\mathbb{R}^d)^n, \quad f_{G,i}(p_1, \dots, p_n) = \mathsf{fti}(p, p_{\text{goal}}, \mathcal{X}) - p$$

Now, there are no topological changes in f_G, hence f_G is continuous

Define set-valued map $T_{\mathsf{CC}} : (\mathbb{R}^d)^n \to \mathbb{P}((\mathbb{R}^d)^n)$

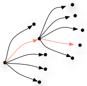
$$T_{\mathsf{CC}}(p_1, \dots, p_n) = \{f_G(p_1, \dots, p_n) \mid G \text{ connected}\}$$



Non-deterministic dynamical systems

Given $T : X \to \mathbb{P}(X)$, a **trajectory** of T is sequence $\{x_m\}_{m \in \mathbb{Z}_{\geq 0}} \subset X$ such that

$$x_{m+1} \in T(x_m), \quad m \in \mathbb{Z}_{\geq 0}$$



T is **closed** at x if $x_m \to x$, $y_m \to y$ with $y_m \in T(x_m)$ imply $y \in T(x)$
Every continuous map $T : \mathbb{R}^d \to \mathbb{R}^d$ is closed on $\mathbb{R}^d$

A set C is

- **weakly positively invariant** if, for any $p_0 \in C$, there exists $p \in T(p_0)$ such that $p \in C$
- **strongly positively invariant** if, for any $p_0 \in C$, all $p \in T(p_0)$ verifies $p \in C$

A point p_0 is a *fixed point of T* if $p_0 \in T(p_0)$

LaSalle Invariance Principle – set-valued maps

$V : X \to \mathbb{R}$ is non-increasing along T on $S \subset X$ if

$$V(x') \leq V(x) \text{ for all } x' \in T(x) \text{ and all } x \in S$$

Theorem (LaSalle Invariance Principle)

For S compact and strongly invariant with V continuous and non-increasing along closed T on S

Any trajectory starting in S converges to largest weakly invariant set contained in $\{x \in S \mid \exists x' \in T(x) \text{ with } V(x') = V(x)\}$

Correctness – T_{CC} is closed

Recall set-valued map $T_{\mathsf{CC}} : (\mathbb{R}^d)^n \to \mathbb{P}((\mathbb{R}^d)^n)$

$$T_{\mathsf{CC}}(p_1, \dots, p_n) = \{f_G(p_1, \dots, p_n) \mid G \text{ connected}\}$$

T_{CC} is **closed**: finite combination of individual continuous maps

In addition,

$$\mathsf{co}(P') \subset \mathsf{co}(P)$$

for all $P' \in T_G(P)$ and $P \in (\mathbb{R}^d)^n$

Correctness – diameter as non-increasing function

$V_{\text{diam}} = \mathsf{diam} \circ \mathsf{co} : (\mathbb{R}^d)^n \to \overline{\mathbb{R}}_+$, by

$$V_{\text{diam}}(P) = \mathsf{diam}(\mathsf{co}(P)) = \max\{\|p_i - p_j\| \mid i, j \in \{1, \dots, n\}\}$$

Let $\mathsf{diag}((\mathbb{R}^d)^n) = \{(p, \dots, p) \in (\mathbb{R}^d)^n \mid p \in \mathbb{R}^d\}$

Lemma

The function $V_{\text{diam}} = \mathsf{diam} \circ \mathsf{co} : (\mathbb{R}^d)^n \to \overline{\mathbb{R}}_+$ verifies:

1. *V_{diam} is continuous and invariant under permutations;*
2. *$V_{\text{diam}}(P) = 0$ if and only if $P \in \mathsf{diag}((\mathbb{R}^d)^n)$;*
3. *V_{diam} is non-increasing along T_{CC}*

Correctness via LaSalle Invariance Principle

To recap

1. T_{CC} is closed
2. $V = \mathsf{diam}$ is non-increasing along T_{CC}
3. Evolution starting from P_0 is contained in $\mathsf{co}(P_0)$ (compact and strongly invariant)

Application of **LaSalle Invariance Principle:** trajectories starting at P_0 converge to M, largest weakly positively invariant set contained in

$$\{P \in \mathsf{co}(P_0) \mid \exists P' \in T_{\mathsf{CC}}(P) \text{ such that } \mathsf{diam}(P') = \mathsf{diam}(P)\}$$

Have to **identify** M! Ideally, $M = \mathsf{diag}((\mathbb{R}^d)^n) \cap \mathsf{co}(P_0)$

Clearly $\mathsf{diag}((\mathbb{R}^d)^n) \cap \mathsf{co}(P_0) \subset M$ – other inclusion by contradiction

LaSalle Invariance Principle – identifying M

Assume $P \in M \setminus (\mathsf{diag}((\mathbb{R}^d)^n) \cap \mathsf{co}(P_0))$, and thus $\mathsf{diam}(P) > 0$
Let G be a connected directed graph and consider $T_G(P)$

1. All non vertices of $\mathsf{co}(P)$ remain in $\mathsf{co}(P) \setminus \text{vertices}(\mathsf{co}(P))$
2. Argument has to be extended to the case where there is more than one agent at a vertex

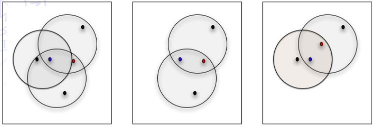
After a finite number of iterations, all agents in configuration $T_{G_1,r}(T_{G_2,r}(\dots T_{G_N,r}(P)))$ are contained in $\mathsf{co}(P) \setminus \mathcal{V}(\mathsf{co}(P))$
Therefore, $\mathsf{diam}(T_{G_1,r}(T_{G_2,r}(\dots T_{G_N,r}(P)))) < \mathsf{diam}(P)$, which contradicts M weakly invariant

Convergence to a point can be concluded with a little bit of extra work

Corollary: Circumcenter algorithm achieves rendezvous

Robustness of circumcenter algorithms

Push whole idea further!, e.g., for robustness against link failures



topology G_1 topology G_2 topology G_3

Look at **evolution under link failures** as outcome of nondeterministic evolution under multiple interaction topologies

$$P \longrightarrow \{\text{evolution under } G_1, \text{evolution under } G_2, \text{evolution under } G_3\}$$

Rendezvous

Corollary (Circumcenter algorithm over $G_{\text{disk}}(r)$ on $\mathbb{R}^d$)

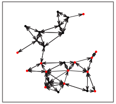
For $\{P_m\}_{m \in \mathbb{Z}_{\geq 0}}$ synchronous execution with link failures such that union of any $\ell \in \mathbb{N}$ consecutive graphs in execution has globally reachable node

Then, there exists $(p^*, \dots, p^*) \in \mathsf{diag}((\mathbb{R}^d)^n)$ such that

$$P_m \to (p^*, \dots, p^*) \quad \textit{as} \quad m \to +\infty$$

Proof uses

$T_{\mathsf{CC},\ell}(P) = \{f_{G_\ell} \circ \dots \circ f_{G_1}(P) \mid \cup_{s=1}^{\ell} G_i \text{ has globally reachable node}\}$



Rendezvous: example complexity analysis

1. first-order agents with disk graph, for $d = 1$,
$$\mathsf{TC}(\mathcal{T}_{\text{rndzvs}}, \mathcal{CC}_{\text{CIRCUMCENTER}}) \in \Theta(n)$$
2. first-order agents with limited Delaunay graph, for $d = 1$,
$$\mathsf{TC}(\mathcal{T}_{(r\varepsilon)\text{-rndzvs}}, \mathcal{CC}_{\text{CIRCUMCENTER}}) \in \Theta(n^2 \log(n\varepsilon^{-1}))$$

Complexity analysis via tridiagonal Toeplitz and circulant matrices

Summary and conclusions

Rendezvous objective

1. Discussed possible algorithms to achieve rendezvous for different networks
2. Constraints to maintain connectivity
3. Results on time complexity
4. Analyzed convergence via nondeterministic dynamical systems
5. Established robustness properties

Set of ideas can be further developed to provide broadly applicable tools for correctness and robustness analysis beyond rendezvous

References

Circumcenter algorithms:

- H. Ando, Y. Oasa, I. Suzuki, and M. Yamashita. Distributed memoryless point convergence algorithm for mobile robots with limited visibility. *IEEE Transactions on Robotics and Automation*, 15(5):818--828, 1999
- J. Cortés, S. Martínez, and F. Bullo. Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. *IEEE Transactions on Automatic Control*, 51(8):1289--1298, 2006

Robustness via non-deterministic dynamical systems:

- J. Cortés. Characterizing robust coordination algorithms via proximity graphs and set-valued maps. In *American Control Conference*, pages 8--13, Minneapolis, MN, June 2006

Flocking algorithms:

- A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6):988--1001, 2003
- L. Moreau. Stability of multiagent systems with time-dependent communication links. *IEEE Transactions on Automatic Control*, 50(2):169--182, 2005